MS455/555 Assignment 2

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Marks

Each part of each task is worth 10 marks. Clearly justify and explain your answers.

Groups

This assignment is to be completed in groups of 2–3 students. Each student should notify the lecturer of their group choice by email by the 23rd of October. Students who do not specify a group preference will be grouped together on a random basis.

Task 1

The p.d.f of the Weibull distribution is given by

$$f(x) = \begin{cases} \alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}}, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

where α and β are positive parameters.

Please present the solutions to the following tasks (and any associated MATLAB code) clearly and succinctly in PDF format:

(a.) The derivation of the cumulative distribution function (c.d.f) F and the inverse c.d.f F^{-1} of a Weibull r.v.

(b.) With $\alpha = \beta = 2$, generate samples from the Weibull distribution using the Inverse transform method ($N = 10, 50, 100, 1000$ for example). Plot the empirical density of each sample and compare it with the true density.

(c.) Perform a Chi-square test on samples of Weibull random variables generated via the Inverse Transform Method. Neatly tabulate your results and explain what you can conclude from your tests.

(d.) For a Weibull distributed r.v. X, calculate the theoretical mean μ and variance σ^2 of X.

(e.) Once more taking $\alpha = \beta = 2$, use μ and σ^2 to write down the 99% asymptotic confidence interval for the sample mean of N Weibull distributed random variables. Explain the theoretical justification for this asymptotic confidence interval in your own words.

Task 2

Suppose the density of a “Polynormal” random variable is given by

$$f(x) = \frac{|x|^3}{4} e^{\frac{-x^2}{2}}, \quad x \in \mathbb{R}.$$

Your task is to use the Acceptance–Rejection method to sample from the “Polynormal” distribution assuming that you have a generator of normal random variables with zero mean and standard deviation $\sigma > 1$.

Please present the solutions to the following tasks (and any associated MATLAB code) clearly and succinctly in PDF format:

(a.) Check that $f(x)$ is a reasonable probability density function. HINT: What are the properties of a p.d.f?

(b.) Plot the p.d.f of the “Polynormal” distribution and the p.d.f of the (zero mean) Normal distribution (for $\sigma = 1.5, 3, 4.5$) on the same axes. Is the Normal distribution a good candidate for the Acceptance–Rejection method on the basis of these plots?

(c.) Denote by $g(x)$ the density of a Normal distribution with mean zero and standard deviation $\sigma > 1$. For a fixed value of $\sigma > 1$, calculate the number $c(\sigma) := \sup_{x \in \mathbb{R}} f(x)/g(x)$.

(d.) Calculate the number $c := \inf_{\sigma > 1} c(\sigma)$.

(e.) Using the above calculations sample from the “Polynormal” distribution via the Acceptance–Rejection method. Provide plots of the empirical density function overlaid with the true density $f(x)$ for various sample sizes.