MS455/555 Assignment 3

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Marks

There are 15 marks for quality and clarity of presentation; the marks for each task are listed below. The assignment is worth 100 marks in total.

Problem Setup

Under a risk–neutral measure, a stock price process $\{S_t : t \geq 0\}$ has dynamics given by the Heston stochastic volatility model, i.e.

$$\begin{aligned} dS_t &= rS_t\,dt + \sqrt{V_t}S_t\,dB_t^S, \qquad && t > 0, \\ dV_t &= \kappa(\theta - V_t)\,dt + \xi\sqrt{V_t}\,dB_t^V, && t > 0, \end{aligned}$$

where $\{V_t : t \geq 0\}$ is the volatility process, r is the risk–free rate, and B_t^S and B_t^V are Brownian motions which are correlated; denote their correlation coefficient by ρ.

Our goal is to price an option with S as the underlying given the following market model parameters:

S_0	V_0	r	ξ	θ	κ	ρ
100	0.40	0.01	0.35	0.04	3.50	-0.40

The option in question matures at time $T = 1$ and has a fixed strike price of $K = 110$. Each group has been given one of the following **path–dependent** options to price:

- Asian call option: Payoff $= \max(A(T) - K, 0)$, where $A(T)$ is the arithmetic mean of S over $[0, T]$.
- Down and out barrier call option: Payoff $= \max(S_T - K, 0)\,\mathbb{1}_{\{\min_{t\in[0,T]} S_t > B\}}$, where $B = 95$ is the barrier level.
- Lookback call option: Payoff $= \max\big(\max_{t\in[0,T]} S_t - K, 0\big)$.

Tasks

(a.) (i.) Briefly comment on the most important similarities and differences between the standard Black–Scholes model and the Heston model.

[5 marks]

(ii.) Explain in your own words why **both** strong and weak convergence are useful concepts for quantifying how well a discrete–time stochastic process approximates a continuous process. Hence explain which of the Euler–Maruyama and Milstein discretisation schemes is most appropriate for this project.

[5 marks]

(b.) (i.) Write a function **HestonEM** which takes as input M (number of paths) and $N = Th$ (where h is the stepsize), and outputs an $N \times M$ matrix in which each column is a simulated path of the stock price process $\{S_t : t \geq 0\}$ in the market described above using the Euler–Maruyama scheme. Explain and justify any corrections or adjustments you make to the standard Euler–Maruyama scheme.

[15 marks]

(ii.) Write a function **HestonMil** which takes as input M (number of paths) and N (as above), and outputs an $N \times M$ matrix in which each column is a simulated path of the stock price process $\{S_t : t \geq 0\}$ in the market described above using the Milstein scheme. Explain and justify any corrections or adjustments you make to the standard Milstein scheme.

[5 marks]

(c.) (i.) Use both of the functions you have written in part (b.) to price your given option for various values of M and N (small to large) using the Monte Carlo estimator. Tabulate your results neatly and briefly comment on them. Briefly explain (with reference to relevant theoretical results) why your code is giving the **fair option prices** in this market.

[40 marks]

(ii.) Calculate 95% and 99% (asymptotic) confidence intervals for the option prices you have calculated in (c.) part (i.). Briefly comment on the computational approach you have used to derive the aforementioned confidence intervals.

[15 marks]