DUBLIN CITY UNIVERSITY

SAMPLE EXAMINATIONS 2017/2018

MODULE:	Simulation for Finance MS555
QUALIFICATIONS:	M.Sc. Financial Mathematics MFM
YEAR OF STUDY:	1
EXAMINERS:	Mr Denis Patterson (ext. 7710), External Examiner
TIME ALLOWED:	2.5 hours
INSTRUCTIONS:	Answer ALL 4 questions
REQUIREMENTS:	Approved calculators and Log tables

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The use of programmable or text storing calculators is expressly forbidden. Please note that where a candidate answers more than the required number of questions, the examiner will mark all questions attempted and then select the highest scoring ones.

QUESTION 1

(a.) A Linear Congruential Generator is a sequence $\{u_i\}_{i\geq 0}$ generated by an iteration of the form

$$x_{i+1} = (ax_i + c) \mod m, \quad i \in \{0, 1, \ldots\},$$
$$u_{i+1} = \frac{x_{i+1}}{m} \in (0, 1), \quad i \in \{0, 1, \ldots\}, \quad a, c, m \in \mathbb{Z}^+.$$

Does a Linear Congruential Generator with $a = 6$, $c = 0$ and $m = 11$ have a full period?

[10 marks]

(b.) $\mathcal{S} = \{x_1, \ldots, x_n\}$ is a sample of uniform random numbers on $[0, 1]$. **Explain** how you would perform a Kolmogorov–Smirnov test for uniformity on the sample $\mathcal{S}$.

[6 marks]

(c.) A sample of 100 pseudorandom numbers which are (supposedly) distributed uniformly on $[0, 1]$ was generated. The interval $[0, 1]$ was divided into 10 equally spaced subintervals and the number of pseudorandom numbers falling into each subinterval was recorded (see the table below).

Table 1: The first (resp. third) row indicates the subinterval and the second (resp. fourth) row indicates the number of sample values in that subinterval.

$[0, 0.1]$	$[0.1, 0.2]$	$[0.2, 0.3]$	$[0.3, 0.4]$	$[0.4, 0.5]$
5	12	12	9	15
$[0.5, 0.6]$	$[0.6, 0.7]$	$[0.7, 0.8]$	$[0.8, 0.9]$	$[0.9, 1]$
10	13	12	7	5

Perform a Chi–Square test for uniformity on the sample considered above at the 95% confidence level.

[9 marks]

HINT: Clearly state the null and alternative hypotheses, and whether or not the null hypothesis was accepted or rejected.

[Total – 25 marks]

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QUESTION 2

(a.) Outline the algorithm for the Inverse Transform Method and prove that it produces random variables with the correct distribution.

[8 marks]

(b.) Consider a one–sided Normal distribution with density

$$f(x) = \sqrt{\frac{2}{\pi\sigma^2}}\, e^{-x^2/2\sigma^2}, \quad x \geq 0, \quad \sigma > 0,$$

and assume that you can sample from an exponential distribution with any parameter $\lambda > 0$.

Which exponential distribution yields the most efficient Acceptance–rejection scheme for the Normal distribution given above?

[12 marks]

HINT: The p.d.f. of an exponential random variable with parameter $\lambda > 0$ is given by

$$g(x) = \lambda e^{-\lambda x}, \quad x \geq 0.$$

(c.) From your calculations in part (b.), on average how many exponential random variables would you need to generate to generate 100 Normal random variables?

[5 marks]

[Total – 25 marks]

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QUESTION 3

Consider the definite integral

$$I = \int_0^\infty x^2 e^{-x}\, dx.$$

(a.) Explain how you would estimate the integral above by interpreting it as the expectation of a Weibull distributed random variable. You should clearly state the Monte Carlo estimator you would use for I.

[10 marks]

HINT: The p.d.f. of a Weibull random variable with parameters $k > 0$ and $\lambda > 0$ is given by

$$f(x) = \begin{cases} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}, & x \geq 0, \\ 0, & x < x_0. \end{cases}$$

(b.) Calculate the mean and variance of the Monte Carlo estimator of I and use the Central Limit Theorem to give an asymptotic 95% confidence interval for the Monte Carlo estimate of I.

[10 marks]

(c.) With explicit reference to relevant theoretical results, explain why the Monte Carlo estimator used above is a good choice of estimator for I.

[5 marks]

[Total – 25 marks]

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QUESTION 4

Consider an autonomous scalar stochastic differential equation of the form

$$dX_t = a(X_t)\,dt + b(X_t)\,dB_t, \quad t \geq 0. \qquad \text{(SDE)}$$

(a.) State precisely sufficient conditions on the functions a and b for (SDE) to have a unique solution.

[6 marks]

(b.) Derive the Euler–Maruyama discretisation scheme for (SDE) and outline its most important performance characteristics. You should define any terminology you introduce in your explanation.

[10 marks]

(c.) Which of the Euler–Maruyama and Milstein would you consider more appropriate for pricing Asian options? Justify your answer.

[3 marks]

(d.) Let $\{B_t, \mathcal{F}_t : t \geq 0\}$ be a standard Brownian motion and suppose the process Y obeys the stochastic differential equation

$$dY_t = \theta dB_t - \frac{1}{2}\theta^2\,dt, \quad t \geq 0; \quad Y_0 = 0,$$

for some $\theta \in \mathbb{R}/\{0\}$. Use Itô's Lemma to derive the stochastic differential equation satisfied by $X_t = e^{Y_t}$ for $t \geq 0$ and hence show that $\mathbb{E}[X_t] = 1$ for each $t \geq 0$.

[6 marks]

[Total – 25 marks]