

## MS455/555 EXTRA EXERCISES ON ITÔ'S LEMMA

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Throughout these exercises  $B = \{B_t : t \geq 0\}$  denotes a standard Brownian motion on some complete filtered probability space  $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ .

### Exercise 1

Consider the nonautonomous SDE given by

$$dX_t = t X_t dt + e^{t^2/2} dB_t, \quad t \geq 0; \quad X_0 = 0.$$

Show that the solution to this SDE is given by

$$X_t = B_t e^{t^2/2}, \quad t \geq 0.$$

### Exercise 2

Suppose that  $X = \{X_t : t \geq 0\}$  is a process which obeys the SDE

$$dX_t = \mu dt + \sigma dB_t, \quad t \geq 0,$$

where  $\mu$  and  $\sigma$  are constants. Derive the SDE obeyed by the process  $Y_t = e^{X_t}$ .

### Exercise 3

Consider the linear SDE given by

$$dX_t = -\alpha X_t dt + \sigma dB_t, \quad t \geq 0,$$

where  $\alpha$  and  $\sigma$  are constants. Show that the solution to this SDE is given by

$$X_t = X_0 e^{-\alpha t} + \sigma \int_0^t e^{-\alpha(t-s)} dB_s, \quad t \geq 0.$$